Mr. Fitch's Math Classes Extended Spring Break Packet 3 (May 6-22, 2020)

ALL CLASSES: Continue to complete your journals, same requirements as the first two packets. Feel free to utilize this journal for other class requirements but make sure you meet the specific requirements for your "math" journal. The requirements for the "math" journal are as follows:

- Five (5) dated entries per week minimum, 10 entries minimum for the two weeks.
- Neatly organized with proper spelling, grammar, and punctuation, bullet points are fine.
- Include any math you used/saw in at least two (2) entries per week.

Each class has assignments listed below, use your time wisely (15-20 minutes per day) and spread these out over the next two weeks, don't wait until the last minute, all assignments will be due when we hopefully return or the next scheduled drop-off date. Answer all questions to the best of your abilities and use your resources to research and investigate. Complete the assignments in the order listed in order to best understand the material. Feel free to email me any questions you may have and I will get back to you as soon as possible (brandon.fitch@leonagroup.com or brandon.fitch@wildwoodisgreen.com).

Online Office Hours (ZOOM): Thursdays 3:00pm-3:50pm: You are not required to attend these.

ALGEBRA 1: (aox45os)

- Lessons 3 and 4: Advanced Factoring Strategies for Quadratic Expressions– Complete Notes and Problem Sets
- Lesson 5: The Zero Product Property – Complete Notes and Problem Set
- Solving Quadratic Equations by Factoring Worksheet – Use what you learned in the notes to complete the worksheet – SHOW ALL WORK

PRECALCULUS (tuhoisb)

- Chapter 4 Test and Trigonometric Functions Test – SHOW ALL WORK

ALGEBRA 2: (e5qngje)

- Lesson 39: Factoring Extended to the Complex Realm – Complete Notes and Problem Set
- Lesson 40: Obstacles Resolved – A surprising Result – Complete Notes and Problem Set
- Complex Numbers and Quadratics Worksheet – Use what you learned in the notes to complete the worksheet – SHOW ALL WORK

GEOMETRY: (m562cco)

- Lessons 25: Incredibly Useful Ratios - Complete Notes and Problem Set
- Trig Functions Powerpoint – Read the notes and answer the questions on the slides
- Lesson 26: Definition of Sine Cosine and Tangent – Complete the Notes and Problem Set
- Trigonometric Ratios Worksheet – Use what you learned in the notes to complete the worksheet.

These assignments will take us through the remainder of the school year. Thank you all for a fun first year at WEA. Seniors, best of luck in all you do and don't hesitate to reach out in the future. Underclassmen, I look forward to seeing you all in the fall when we can hopefully return to a normal school year. Everyone, get this packet done and start enjoying your summer!

Mr Fitch

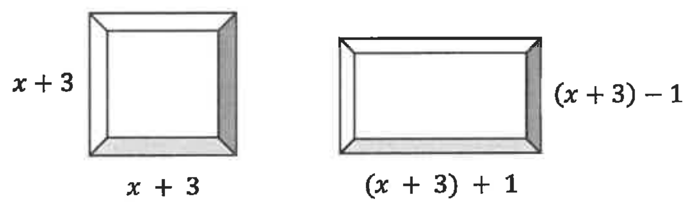
Mr Fitch

Lesson 3: Advanced Factoring Strategies for Quadratic Expressions

Classwork

Opening Exercise

Carlos wants to build a sandbox for his little brother. He is deciding between a square sandbox with side lengths that can be represented by $x + 3$ units and a rectangular sandbox with a length 1 unit more than the side of the square and width 1 unit less than the side of the square.



Carlos thinks the areas should be the same because one unit is just moved from one side to the other.

a. Do you agree that the two areas should be the same? Why or why not?

b. How would you write the expressions that represent the length and width of the rectangular sandbox in terms of the side length of the square?

c. If you use the expressions for length and width represented in terms of the side length of the square, can you then write the area of the rectangle in the same terms?

d. How can this expression be seen as the product of a sum and difference: $(a+b)(a-b)$?

e. Can you now rewrite the area expression for the rectangle as the difference of squares: $(a+b)(a-b)=a^2-b^2$?

f. Look carefully at your answer to the last question. What does it tell you about the areas of the two shapes?

g. Can you verify that our algebra is correct using a diagram or visual display?

Example 1

In Lesson 2, we saw that factoring is the reverse process of multiplication. We factor a polynomial by reversing the distribution process.

Consider the following example of multiplication:

$$(x+3)(x+5) \rightarrow x^2+5x+3x+15 \rightarrow x^2+8x+15.$$

When we compare the numbers in the factored form with the numbers in the expanded form, we see that 15 is the product of the two numbers $(3 \cdot 5)$, and 8 is their sum $(3+5)$. The latter is even more obvious when we look at the expanded form before the like terms are combined.

Can you explain why that relationship exists between the numbers in the factors and the numbers in the final expanded form?

Example 2

Now compare the expansion of this binomial product to the one above:

$$(2x+3)(1x+5) \rightarrow 2x^2+10x+3x+15 \rightarrow 2x^2+13x+15.$$

In the expression lying between the two arrows (before the like terms are combined), we can see the coefficients of the "split" linear terms $(+10x+3x)$. Also notice that for this example, we have coefficients on both x-terms in the factors and that one of the coefficients is not 1. We have 2 and 1 as the factors of the leading coefficient in the expanded form and 3 and 5 as the factors of the constant term. Get ready for quadratic expressions in factored form where neither of the x-term coefficients are 1.

a. How is this product different from the first example? How is it similar?

b. Why are the "split" linear terms different in the two examples?

c. Now that we have four different numbers (coefficients) in each form of the expression, how can we use the numbers in the expanded form of the quadratic expression on the right to find the numbers in the factors on the left?

d. Now we need to place those numbers into the parentheses for the factors so that the product matches the expanded form of the quadratic expression. Here is a template for finding the factors using what we call the product-sum method:

(___x $\pm$ __)(___x $\pm$ ___) [We have four number places to fill in this factor template.]

(___x $\pm$ 3)(___x $\pm$ 5) [We know that the 3 and 5 are the correct factors for 15, so we start there.]

(2 x $\pm$ 3)(1 x $\pm$ 5) [We know that 2 and 1 are the only factors of 2, with the 2 opposite the 5 so that the distribution process gives us $10x$ for one product.]

($2x$ + 3)(x + 5) [Finally, we know, at least for this example, that all the numbers are positive.]

Example 3

Now try factoring a quadratic expression with some negative coefficients: $3x^2 - x - 4$.

(___x $\pm$ ___)(___x $\pm$ ___) [We have four number places to fill in this factor template.]

(___x $\pm$ 1)(___x $\pm$ 4) [We know that ± 1 and ± 4 or ± 2 and ± 2 are the only possible factors for the constant term, -4, so we start there. Try 1 and 4 to start, and if that does not work, go back and try ± 2 and ± 2. We know that only one of the numbers can be negative to make the product negative.]

(1 x $\pm$ 1)(3 x $\pm$ 4) [We know that 3 and 1 are the only factors of 3. We also know that both of these are positive (or both negative). But we do not know which positions they should take, so we will try both ways to see which will give a sum of -1.]

(x + 1)($3x - 4$) [Finally, we determine the two signs needed to make the final product $3x^2 - x - 4$.]

Exercises

For Exercises 1–6, factor the expanded form of these quadratic expressions. Pay particular attention to the negative and positive signs.

1. $3x^2 - 2x - 8$

2. $3x^2 + 10x - 8$

3. $3x^2 + x - 14$ [Notice that there is a 1 as a coefficient in this one.]

4. $2x^2 - 21x - 36$ [This might be a challenge. If it takes too long, try the next one.]

5. $-2x^2 + 3x + 9$ [This one has a negative on the leading coefficient.]

6. $r^2 + \frac{6}{4}r + \frac{9}{16}$ [We need to try one with fractions, too.]

For Exercises 7–10, use the structure of these expressions to factor completely.

7. $100x^2 - 20x - 63$

8. $y^4 + 2y^2 - 3$

9. $9x^2 - 3x - 12$

10. $16a^2b^4 + 20ab^2 - 6$

Lesson Summary

QUADRATIC EXPRESSION: A polynomial expression of degree 2 is often referred to as a *quadratic expression*.

Some quadratic expressions are not easily factored. The following hints will make the job easier:

- In the difference of squares $a^2 - b^2$, either of these terms a or b could be a binomial itself.
- The product-sum method is useful but can be tricky when the leading coefficient is not 1.
- Trial and error is a viable strategy for finding factors.
- Check your answers by multiplying the factors to ensure you get back the original quadratic expression.

Problem Set

Factor the following quadratic expressions.

1. $x^2 + 9x + 20$
2. $3x^2 + 27x + 60$
3. $4x^2 + 9x + 5$
4. $3x^2 - 2x - 5$
5. $-2x^2 + 5x$
6. $-2x^2 + 5x - 2$
7. $5x^2 + 19x - 4$
8. $4x^2 - 9$
9. $4x^2 - 12x + 9$ [This one is tricky, but look for a special pattern.]
10. $3x^2 - 13x + 12$

Factor each expression completely.

11. $a^4 - b^4$

12. $16a^4 - b^4$

13. $a^2 - 5a + 4$

14. $a^4 - 5a^2 + 4$

15. $9a^2 - 15a + 4$

Lesson 4: Advanced Factoring Strategies for Quadratic Expressions

Classwork

Opening Exercise

Factor the following quadratic expressions.

a. $2x^2 + 10x + 12$

b. $6x^2 + 5x - 6$

Example: Splitting the Linear Term

How might we find the factors of $6x^2 + 5x - 6$?

1. Consider the product $(a)(c)$: $(6)(-6) = -36$.
2. Discuss the possibility that a and c are also multiplied when the leading coefficient is 1.
3. List all possible factor pairs of $(a)(c)$: $(1, -36)$, $(-1, 36)$, $(2, -18)$, $(-2, 18)$, $(3, -12)$, $(-3, 12)$, $(4, -9)$, $(-4, 9)$, and $(-6, 6)$.
4. Find the pair that satisfies the requirements of the product-sum method (i.e., a pair of numbers whose product equals ac and whose sum is b): $(-4) + 9 = 5$.
5. Rewrite the expression with the same first and last term but with an expanded b term using that pair of factors as coefficients: $6x^2 - 4x + 9x - 6$.
6. We now have four terms that can be entered into a tabular model or factored by grouping.
7. Factoring by grouping: Take the four terms above and pair the first two and the last two; this makes two *groups*.

   $[6x^2 - 4x] + [9x - 6]$ [Form two groups by pairing the first two and the last two.]

   $[2x(3x - 2)] + [3(3x - 2)]$ [Factor out the GCF from each pair.]

   The common binomial factor is now visible as a common factor of each group. Now rewrite by carefully factoring out the common factor, $3x - 2$, from each group: $(3x - 2)(2x + 3)$.

Note that we can factor difficult quadratic expressions, such as $6x^2 + 5x - 6$, using a tabular model or by splitting the linear term algebraically. Try both ways to see which one works best for you.

Exercise

Factor the following expressions using your method of choice. After factoring each expression completely, check your answers using the distributive property. Remember to always look for a GCF prior to trying any other strategies.

1. $2x^2 - x - 10$

2. $6x^2 + 7x - 20$

3. $-4x^2 + 4x - 1$

4. The area of a particular triangle can be represented by $x^2 + \frac{3}{2}x - \frac{9}{2}$. What are its base and height in terms of x?

Lesson Summary

While there are several steps involved in splitting the linear term, it is a relatively more efficient and reliable method for factoring trinomials in comparison to simple guess-and-check.

Problem Set

1. Factor completely.

   a. $9x^2 - 25x$

   b. $9x^2 - 25$

   c. $9x^2 - 30x + 25$

   d. $2x^2 + 7x + 6$

   e. $6x^2 + 7x + 2$

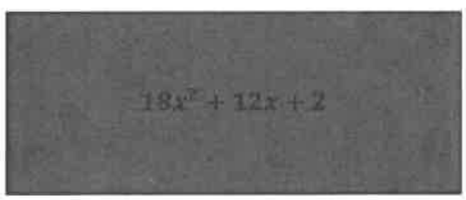
   f. $8x^2 + 20x + 8$

   g. $3x^2 + 10x + 7$

   h. $x^2 + \frac{11}{2}x + \frac{5}{2}$

   i. $6x^3 - 2x^2 - 4x$ [Hint: Look for a GCF first.]

2. The area of the rectangle below is represented by the expression $18x^2 + 12x + 2$ square units. Write two expressions to represent the dimensions, if the length is known to be twice the width.



3. Two mathematicians are neighbors. Each owns a separate rectangular plot of land that shares a boundary and has the same dimensions. They agree that each has an area of $2x^2 + 3x + 1$ square units. One mathematician sells his plot to the other. The other wants to put a fence around the perimeter of his new combined plot of land. How many linear units of fencing does he need? Write your answer as an expression in x.

   Note: This question has two correct approaches and two different correct solutions. Can you find them both?

Lesson 5: The Zero Product Property

Classwork

Opening Exercise

Consider the equation $a \cdot b \cdot c \cdot d = 0$. What values of a, b, c, and d would make the equation true?

Exercises 1–4

Find values of c and d that satisfy each of the following equations. (There may be more than one correct answer.)

1. $cd = 0$

2. $(c - 5)d = 2$

3. $(c-5)d=0$

4. $(c-5)(d+3)=0$

Example 1

For each of the related questions below, use what you know about the zero product property to find the answers.

a. The area of a rectangle can be represented by the expression x^2+2x-3. If the dimensions of the rectangle are known to be the linear factors of the expression, write each dimension of this rectangle as a binomial. Write the area in terms of the product of the two binomials.

b. Draw and label a diagram that represents the rectangle's area.

c. Suppose the rectangle's area is 21 square units. Can you find the dimensions of the rectangle?

d. Rewrite the equation so that it is equal to zero and solve.

e. What are the actual dimensions of the rectangle?

f. A smaller rectangle can fit inside the first rectangle, and it has an area that can be represented by the expression $x^2 - 4x - 5$. If the dimensions of the rectangle are known to be the linear factors of the expression, what are the dimensions of the smaller rectangle in terms of x?

g. What value for x would make the smaller rectangle have an area of $\frac{1}{3}$ that of the larger?

Exercises 5–8

Solve. Show your work.

5. $x^2 - 11x + 19 = -5$

6. $7x^2 + x = 0$

7. $7r^2 - 14r = -7$

8. $2d^2 + 5d - 12 = 0$

Lesson Summary

Zero Product Property

If $ab = 0$, then $a = 0$ or $b = 0$ or $a = b = 0$.

When solving for the variable in a quadratic equation, rewrite the quadratic expression in factored form and set equal to zero. Using the zero product property, you know that if one factor is equal to zero, then the product of all factors is equal to zero.

Going one step further, when you have set each binomial factor equal to zero and have solved for the variable, all of the possible solutions for the equation have been found. Given the context, some solutions may not be viable, so be sure to determine if each possible solution is appropriate for the problem.

Problem Set

Solve the following equations.

1. $(2x - 1)(x + 3) = 0$
2. $(t - 4)(3t + 1)(t + 2) = 0$
3. $x^2 - 9 = 0$
4. $(x^2 - 9)(x^2 - 100) = 0$
5. $x^2 - 9 = (x - 3)(x - 5)$
6. $x^2 + x - 30 = 0$
7. $p^2 - 7p = 0$
8. $p^2 - 7p = 8$
9. $3x^2 + 6x + 3 = 0$
10. $2x^2 - 9x + 10 = 0$

11. $x^2 + 15x + 40 = 4$

12. $7x^2 + 2x = 0$

13. $7x^2 + 2x - 5 = 0$

14. $b^2 + 5b - 35 = 3b$

15. $6r^2 - 12r = 18$

16. $2x^2 + 11x = x^2 - x - 32$

17. Write an equation (in factored form) that has solutions of $x = 2$ or $x = 3$.

18. Write an equation (in factored form) that has solutions of $a = 0$ or $a = -1$.

19. Quinn looks at the equation $(x - 5)(x - 6) = 2$ and says that since the equation is in factored form it can be solved as follows:

$$(x - 5)(x - 6) = 2$$
$$x - 5 = 2 \text{ or } x - 6 = 2$$
$$x = 7 \text{ or } x = 8.$$

Explain to Quinn why this is incorrect. Show her the correct way to solve the equation.

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Solving Quadratic Equations by Factoring

Date________________ Period____

Solve each equation by factoring.

1) $(k+1)(k-5)=0$

2) $(a+1)(a+2)=0$

3) $(4k+5)(k+1)=0$

4) $(2m+3)(4m+3)=0$

5) $x^2-11x+19=-5$

6) $n^2+7n+15=5$

7) $n^2-10n+22=-2$

8) $n^2+3n-12=6$

9) $6n^2-18n-18=6$

10) $7r^2-14r=-7$

11) $n^2 + 8n = -15$

12) $5r^2 - 44r + 120 = -30 + 11r$

13) $-4k^2 - 8k - 3 = -3 - 5k^2$

14) $b^2 + 5b - 35 = 3b$

15) $3r^2 - 16r - 7 = 5$

16) $6b^2 - 13b + 3 = -3$

17) $7k^2 - 6k + 3 = 3$

18) $35k^2 - 22k + 7 = 4$

19) $7x^2 + 2x = 0$

20) $10b^2 = 27b - 18$

21) $8x^2 + 21 = -59x$

22) $15a^2 - 3a = 3 - 7a$