Mr. Fitch's Math Classes Extended Spring Break Packet 3 (May 6-22, 2020)

ALL CLASSES: Continue to complete your journals, same requirements as the first two packets. Feel free to utilize this journal for other class requirements but make sure you meet the specific requirements for your "math" journal. The requirements for the "math" journal are as follows:

- Five (5) dated entries per week minimum, 10 entries minimum for the two weeks.
- Neatly organized with proper spelling, grammar, and punctuation, bullet points are fine.
- Include any math you used/saw in at least two (2) entries per week.

Each class has assignments listed below, use your time wisely (15-20 minutes per day) and spread these out over the next two weeks, don't wait until the last minute, all assignments will be due when we hopefully return or the next scheduled drop-off date. Answer all questions to the best of your abilities and use your resources to research and investigate. Complete the assignments in the order listed in order to best understand the material. Feel free to email me any questions you may have and I will get back to you as soon as possible (brandon.fitch@leonagroup.com or brandon.fitch@wildwoodisgreen.com).

Online Office Hours (ZOOM): Thursdays 3:00pm-3:50pm: You are not required to attend these.

ALGEBRA 1: (aox45os)

- Lessons 3 and 4: Advanced Factoring Strategies for Quadratic Expressions– Complete Notes and Problem Sets
- Lesson 5: The Zero Product Property – Complete Notes and Problem Set
- Solving Quadratic Equations by Factoring Worksheet – Use what you learned in the notes to complete the worksheet – SHOW ALL WORK

PRECALCULUS (tuhoisb)

- Chapter 4 Test and Trigonometric Functions Test – SHOW ALL WORK

ALGEBRA 2: (e5qngje)

- Lesson 39: Factoring Extended to the Complex Realm – Complete Notes and Problem Set
- Lesson 40: Obstacles Resolved – A surprising Result – Complete Notes and Problem Set
- Complex Numbers and Quadratics Worksheet – Use what you learned in the notes to complete the worksheet – SHOW ALL WORK

GEOMETRY: (m562cco)

- Lessons 25: Incredibly Useful Ratios - Complete Notes and Problem Set
- Trig Functions Powerpoint – Read the notes and answer the questions on the slides
- Lesson 26: Definition of Sine Cosine and Tangent – Complete the Notes and Problem Set
- Trigonometric Ratios Worksheet – Use what you learned in the notes to complete the worksheet.

These assignments will take us through the remainder of the school year. Thank you all for a fun first year at WEA. Seniors, best of luck in all you do and don't hesitate to reach out in the future. Underclassmen, I look forward to seeing you all in the fall when we can hopefully return to a normal school year. Everyone, get this packet done and start enjoying your summer!

Mr Fitch

Lesson 39: Factoring Extended to the Complex Realm

Classwork

Opening Exercise

Rewrite each expression as a polynomial in standard form.

a. $(x+i)(x-i)$

b. $(x+5i)(x-5i)$

c. $(x-(2+i))(x-(2-i))$

Exercises 1–4

Factor the following polynomial expressions into products of linear terms.

1. x^2+9

2. x^2+5

3. Consider the polynomial $P(x) = x^4 - 3x^2 - 4$.

   a. What are the solutions to $x^4 - 3x^2 - 4 = 0$?

   b. How many x-intercepts does the graph of the equation $y = x^4 - 3x^2 - 4$ have? What are the x-intercepts?

   c. Are solutions to the polynomial equation $P(x) = 0$ the same as the x-intercepts of the graph of $y = P(x)$? Justify your reasoning.

4. Write a polynomial P with the lowest possible degree that has the given solutions. Explain how you generated each answer.
   a. $-2, 3, -4i, 4i$

   b. $-1, 3i$

   c. $0, 2, 1+i, 1-i$

   d. $\sqrt{2}, -\sqrt{2}, 3, 1+2i$

   e. $2i, 3-i$

Lesson Summary

- Polynomial equations with real coefficients can have real or complex solutions or they can have both.
- If a complex number is a solution to a polynomial equation, then its conjugate is also a solution.
- Real solutions to polynomial equations correspond to the x-intercepts of the associated graph, but complex solutions do not.

Problem Set

1. Rewrite each expression in standard form.
   a. $(x+3i)(x-3i)$
   b. $(x-a+bi)(x-(a+bi))$
   c. $(x+2i)(x-i)(x+i)(x-2i)$
   d. $(x+i)^2 \cdot (x-i)^2$

2. Suppose in Problem 1 that you had no access to paper, writing utensils, or technology. How do you know that the expressions in parts (a)–(d) are polynomials with real coefficients?

3. Write a polynomial equation of degree 4 in standard form that has the solutions $i, -i, 1, -1$.

4. Explain the difference between x-intercepts and solutions to an equation. Give an example of a polynomial with real coefficients that has twice as many solutions as x-intercepts. Write it in standard form.

5. Find the solutions to $x^4 - 5x^2 - 36 = 0$ and the x-intercepts of the graph of $y = x^4 - 5x^2 - 36$.

6. Find the solutions to $2x^4 - 24x^2 + 40 = 0$ and the x-intercepts of the graph of $y = 2x^4 - 24x^2 + 40$.

7. Find the solutions to $x^4 - 64 = 0$ and the x-intercepts of the graph of $y = x^4 - 64$.

8. Use the fact that $x^4 + 64 = (x^2 - 4x + 8)(x^2 + 4x + 8)$ to explain how you know that the graph of $y = x^4 + 64$ has no x-intercepts. You need not find the solutions.

Lesson 40: Obstacles Resolved—A Surprising Result

Classwork

Opening Exercise

Write each of the following quadratic expressions as a product of linear factors. Verify that the factored form is equivalent.

a. $x^2 + 12x + 27$

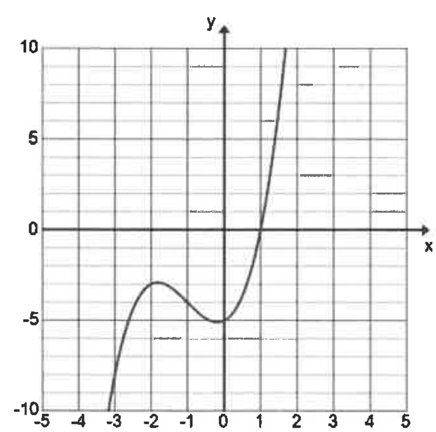
b. $x^2 - 16$

c. $x^2 + 16$

d. $x^2 + 4x + 5$

Example 1

Consider the polynomial $P(x) = x^3 + 3x^2 + x - 5$ whose graph is shown to the right.



a. Looking at the graph, how do we know that there is only one real solution?

b. Is it possible for a cubic polynomial function to have no zeros?

c. From the graph, what appears to be one solution to the equation $x^3 + 3x^2 + x - 5 = 0$?

d. How can we verify that this value is a solution?

e. According to the remainder theorem, what is one factor of the cubic expression $x^3 + 3x^2 + x - 5$?

f. Factor out the expression you found in part (e) from $x^3 + 3x^2 + x - 5$.

g. What are all of the solutions to $x^3 + 3x^2 + x - 5 = 0$?

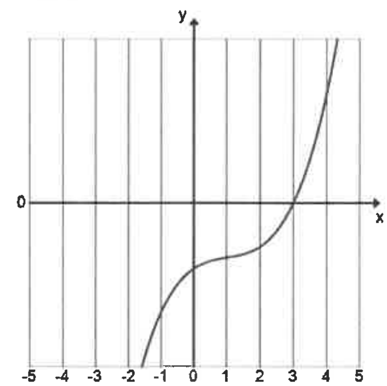
h. Write the expression $x^3 + 3x^2 + x - 5$ in terms of linear factors.

Exercises 1–2

Write each polynomial in terms of linear factors. The graph of $y = x^3 - 3x^2 + 4x - 12$ is provided for Exercise 2.

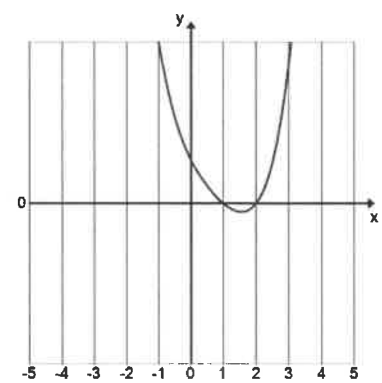
1. $f(x) = x^3 + 5x$

2. $g(x) = x^3 - 3x^2 + 4x - 12$



Example 2

Consider the polynomial function $P(x) = x^4 - 3x^3 + 6x^2 - 12x + 8$, whose corresponding graph $y = x^4 - 3x^3 + 6x^2 - 12x + 8$ is shown to the right. How many zeros does P have?



a. Part 1 of the fundamental theorem of algebra says that this equation will have at least one solution in the complex numbers. How does this align with what we can see in the graph to the right?

b. Identify one zero from the graph.

c. Use polynomial division to factor out one linear term from the expression $x^4 - 3x^3 + 6x^2 - 12x + 8$.

d. Now we have a cubic polynomial to factor. We know by part 1 of the fundamental theorem of algebra that a polynomial function will have at least one real zero. What is that zero in this case?

e. Use polynomial division to factor out another linear term of $x^4 - 3x^3 + 6x^2 - 12x + 8$.

f. Are we done? Can we factor this polynomial any further?

g. Now that the polynomial is in factored form, we can quickly see how many solutions there are to the original equation $x^4 - 3x^3 + 6x^2 - 12x + 8 = 0$.

h. What if we had started with a polynomial function of degree 8?

Lesson Summary

Every polynomial function of degree n, for $n \geq 1$, has n roots over the complex numbers, counted with multiplicity. Therefore, such polynomials can always be factored into n linear factors, and the obstacles to factoring we saw before have all disappeared in the larger context of allowing solutions to be complex numbers.

The Fundamental Theorem of Algebra:

1. If P is a polynomial function of degree $n \geq 1$, with real or complex coefficients, then there exists at least one number r (real or complex) such that $P(r) = 0$.
2. If P is a polynomial function of degree $n \geq 1$, given by $P(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ with real or complex coefficients a_i, then P has exactly n zeros $r_1, r_2, \ldots, r_n$ (not all necessarily distinct), such that $P(x) = a_n(x - r_1)(x - r_2)\cdots(x - r_n)$.

Problem Set

1. Write each quadratic function below in terms of linear factors.
   a. $f(x) = x^2 - 25$
   b. $f(x) = x^2 + 25$
   c. $f(x) = 4x^2 + 25$
   d. $f(x) = x^2 - 2x + 1$
   e. $f(x) = x^2 - 2x + 4$

2. Consider the polynomial function $P(x) = (x^2 + 4)(x^2 + 1)(2x + 3)(3x - 4)$.
   a. Express P in terms of linear factors.
   b. Fill in the blanks of the following sentence.

   The polynomial P has degree ______________ and can, therefore, be written in terms of ________ linear factors. The function P has _________ zeros. There are ___________ real zeros and _____ complex zeros. The graph of $y = P(x)$ has ______ x-intercepts.

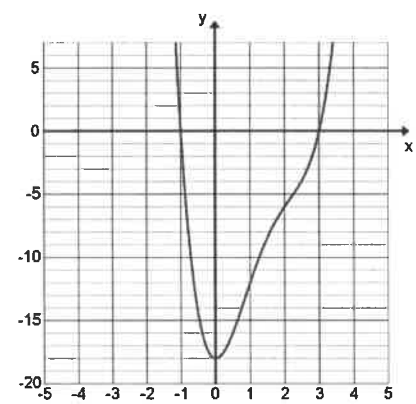
3. Express each cubic function below in terms of linear factors.
   a. $f(x) = x^3 - 6x^2 - 27x$
   b. $f(x) = x^3 - 16x^2$
   c. $f(x) = x^3 + 16x$

4. For each cubic function below, one of the zeros is given. Express each cubic function in terms of linear factors.
   a. $f(x) = 2x^3 - 9x^2 - 53x - 24$; $f(8) = 0$
   b. $f(x) = x^3 + x^2 + 6x + 6$; $f(-1) = 0$

5. Determine if each statement is always true or sometimes false. If it is sometimes false, explain why it is not always true.
   a. A degree 2 polynomial function will have two linear factors.
   b. The graph of a degree 2 polynomial function will have two x-intercepts.
   c. The graph of a degree 3 polynomial function might not cross the x-axis.
   d. A polynomial function of degree n can be written in terms of n linear factors.

6. Consider the polynomial function $f(x) = x^6 - 9x^3 + 8$.
   a. How many linear factors does $x^6 - 9x^3 + 8$ have? Explain.
   b. How is this information useful for finding the zeros of f?
   c. Find the zeros of f. (Hint: Let $Q = x^3$. Rewrite the equation in terms of Q to factor.)

7. Consider the polynomial function $P(x) = x^4 - 6x^3 + 11x^2 - 18$.
   a. Use the graph to find the real zeros of P.
   b. Confirm that the zeros are correct by evaluating the function P at those values.
   c. Express P in terms of linear factors.
   d. Find all zeros of P.



8. Penny says that the equation $x^3 - 8 = 0$ has only one solution, $x = 2$. Use the fundamental theorem of algebra to explain to her why she is incorrect.

9. Roger says that the equation $x^2 - 12x + 36 = 0$ has only one solution, 6. Regina says Roger is wrong and that the fundamental theorem of algebra guarantees that a quadratic equation must have two solutions. Who is correct and why?

College Algebra Quiz Review Name____________________ ID: 1

Complex Numbers and Quadratics

Date________________ Period____

Simplify.

1) $(-3-5i)-(-4-5i)$

2) $(-4i)+(8+7i)+(-2+4i)$

3) $(6-8i)^2$

4) $(2+2i)(-4-7i)$

5) $\frac{7}{3i}$

6) $\frac{4i}{6-3i}$

7) $\frac{2-3i}{2+10i}$

Solve each equation by factoring.

8) $p^2=-11p-28$

9) $35k^2-6=37k$

10) $-7p^2-9p+18=-8p^2$

Solve each equation by taking square roots.

11) $x^2-9=16$

12) $2x^2-3=29$

13) $5p^2+5=495$

14) $r^2-10=-20$

15) $-2-3p^2=-54$

Solve each equation by completing the square.

16) $b^2-12b+35=0$

17) $6x^2+12x-78=0$

Solve each equation with the quadratic formula.

18) $k^2+8=6k$

19) $x^2=-5x+23$